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NATURAL CONVECTION IN AN INCLINED RECTANGULAR CHANNEL AT VARIOUS ASPECT RATIOS AND ANGLES—EXPERIMENTAL MEASUREMENTS

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Abstract—Rates of heat transfer were measured for laminar natural convection in silicone oil and air in a long rectangular channel. The aspect ratio (width/height) of the cross-section of the channel was varied over 1, 2, 3, 4·2, 8·4 and 15·5, and the Rayleigh number from 3×10^3 to 10^5. The channel was heated from below and cooled from above while the other two sides were insulated. The channel was then rotated about the long axis in steps through 180 degrees. The effect of inclination and of the aspect ratio on the rate of heat transfer was measured experimentally.

A minimum and a maximum rate of heat transfer occurred as the angle of inclination was increased from 0 to 180 degrees. The angle of inclination at these critical conditions was found to be a strong function of the aspect ratio and a weak function of the Rayleigh number. A transition in the mode of circulation occurred at the angle corresponding to the minimum rate of heat transfer.

NOMENCLATURE

c_p, specific heat;
g, acceleration due to gravity;
H, height of channel;
k, thermal conductivity;
L, length of channel;
Nu, Nusselt number, $= qH/k(\theta_h-\theta_c)$;
$\overline{Nu}$, average Nusselt numbers, $=(1/W)\int_0^W Nu\,dx$;
Pr, Prandtl number, $= c_p\mu/k$;
q, heat flux density;
Q, total heat input to the lower plate;
Ra, Rayleigh number, $=(\rho^2 g c_p \beta(\theta_h-\theta_c)H^3/k\mu)_0$;
W, width of the channel in the x-direction;
x, distance across hot plate.

Greek symbols

β, volumetric coefficient of expansion with temperature;
θ, temperature;
μ, viscosity;
ρ, density;
ψ, degree of inclination of the hot plate from the horizontal plane.

Subscript

0, value at mean temperature;
h, value at hot plate;
c, value at cold plate.

INTRODUCTION

NATURAL convection in an inclined rectangular region has received increasing attention in recent years. Hart [1, 2] studied experimentally and theoretically the stability of flow in a differentially-heated, inclined, shallow box for water and air, but he did not report the rate of heat transfer.

Unny [3] studied thermal instability in differentially-heated, inclined fluid layers theoretically, and discussed the effect of Prandtl number on the preferred mode of circulation. Hollands and Konicek [4] reported on an experimental study of the stability of differentially-heated, inclined air layers in a shallow box with an aspect ratio of 44. They reported Nusselt numbers near the critical conditions as a function of the Rayleigh number and angle of inclination. Clever [5] reported that the experimental rate of heat transfer can be correlated as a function of $Ra \cos\psi$ where ψ is the angle of inclination of the heated plate from the horizontal plane, but his correlation is only for longitudinal rolls. These studies were all for very shallow boxes and instability was to be expected. Davis [6], Catton [7] and others have reported on the effect of insulated vertical walls on the onset of natural convection in a fluid heated from below. There does not appear to have been a definitive study of the combined effect of aspect ratio and angle of inclination on the circulation and rate of heat transfer in an inclined, finite, rectangular region.

Ozoe, Sayama and Churchill [8] studied natural convection in an inclined long channel with a square cross-section and found a minimum and a maximum in the rate of heat transfer during the rotation of the hot plate from the horizontal to the vertical about the long axis. They integrated a partial differential-equation model for a single roll-cell with its axis in the long dimension of the channel and found good agreement with experiments for the average rate of heat transfer. In a subsequent paper [9] the effect of the aspect ratio of the cross-section of the channel was investigated theoretically for angles of inclination through 180 degrees and $Pr = 10$.*

In the work reported herein the experimental rate of heat transfer was measured in detail and compared with these prior theoretical values for $Pr = 10$. Aspect ratios of 1, 2, 3, 4·2, 8·4 and 15·5, and Rayleigh numbers from 3×10^3 to 10^5 were investigated using silicone oil and air.

EXPERIMENTAL APPARATUS

The experimental apparatus was essentially the same as that described in reference [8] and is shown schematically in Fig. 1. The four vertical sides of the

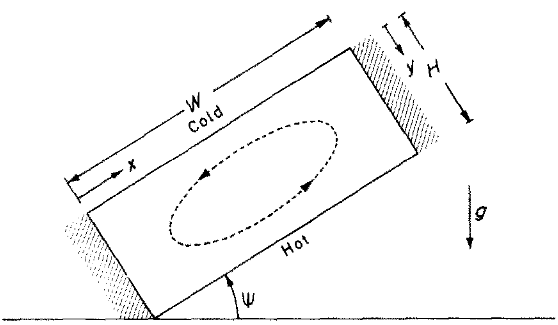

FIG. 1. Configuration of mathematical model and experiments.

channel were made of transparent Plexiglas. The upper and lower sides of the channel were copper plates to assure a uniform temperature distribution. The upper plate was cooled by water circulating through a constant temperature bath. The lower plate was heated by Nichrom wire and the total heat input was measured by a wattmeter. The temperature difference between the upper and lower plates was measured with a copper-constantan thermocouple and a microvolt potentiometer. The dimensions of the channels and the two aspect ratios are listed in the Table 1. When filling

Table 1. Channel dimensions

Nominal aspect ratio	H (cm)	W (cm)	L (cm)	W/H	L/H
1	4·1	3·96	24·00	0·966	5·85
2	2·13	4·00	23·93	1·88	11·23
3	1·98	6·00	24·00	3·03	12·12
4·2	1·90	8·00	24·00	4·22	12·63
8·4	2·15	18·05	18·05	8·40	8·40
15·5	1·16	17·98	17·99	15·5	15·50

with silicone oil, the Plexiglas channel was placed on the lower copper plate; then the upper plate was put in place carefully to avoid air bubbles in the fluid, and the whole apparatus was clamped firmly with nuts and bolts. During the experiments, the entire apparatus was covered with thick glass-wool to prevent heat transfer to and from the surroundings. The experiments were carried out in a room with the temperature maintained at the temperature of the cooling water, thus assuring no heat flux in the absence of a heat input to the lower plate. The most difficult part of the experiment was to detect the net heat flux through the experimental fluid. At 180 degrees of inclination, the fluid was heated from above and the conductive heat flux was estimated from the known thermal conductivity of the fluid. The heat loss was determined by subtracting the computed conductive heat flux from the total heat input. The same heat loss was assumed to occur during the convective heat-transfer experiments. When the data were plotted as the temperature difference between the upper and lower plates vs the total heat input, the data for the conductive heat-transfer regime ($Ra < Ra^{\text{critical}}$) at zero degrees of inclination overlapped the data for 180 degrees of inclination. Since the upper plate was maintained at constant temperature while the lower plate was heated, the mean temperature of the fluid varied slightly as the angle of inclination was increased and the temperature difference between the plates changed. The Rayleigh number was calculated using physical properties at the average of the two plate temperatures. Hence the experimental points for a constant heat input are at slightly different Rayleigh numbers. A new steady state was attained 5–8 h after the angle of inclination was changed. The temperature across the lower plate was ascertained to be uniform at each steady state. The flow pattern in the silicone oil was simulated with aluminum powder in glycerol. Smoke from the combustion of vinyl resin was used with air.

EXPERIMENTAL RESULTS

The experimental rates of heat transfer are plotted for each channel in Figs. 2–6. Silicone oil was used in the channels of aspect ratio $W/H = 1, 2, 3$ and 4·2. Air was used in the channels of aspect ratio $W/H = 8{\cdot}4$ and 15·5. It is presumed that L/H was in all cases sufficiently large (see Table 1) so as not to be a significant variable but this effect has not been investigated. Each group of data points is for the same heat input at various angles from 0 to 180 degrees. For each group of data a minimum and a maximum in $\overline{Nu}$ were observed. When the angle of inclination from the horizontal was zero, the flow pattern was a series of roll-cells with axes parallel to each other and perpendicular to the long axis of the channel.

When the angle was increased slightly in steps, a series of roll-cells persisted, with their axes in the upslope, but $\overline{Nu}$ decreased. When the angle was further increased, a minimum in $\overline{Nu}$ was attained, then an increase. As $\overline{Nu}$ increased the flow pattern switched to a single roll-cell with its axis in the long and horizontal dimension of the channel. This flow pattern persisted as the angle of inclination increased up to 180 degrees (heating from above) and $\overline{Nu}$ went through a maximum

*In [9], ρ_0 should be replaced by ρ in equations (4–6) and H by W/H in the definition of $\overline{Nu}$. Also 165° should be replaced by 100–175° in the discussion of Hart [13] on p. 1210.

and then decreased to unity. This same behavior was observed qualitatively for all of the channels, but the angle of inclination corresponding to the minimum and maximum values of $\overline{Nu}$ depended strongly on the aspect ratio. These critical angles were however almost invariant with Rayleigh number.

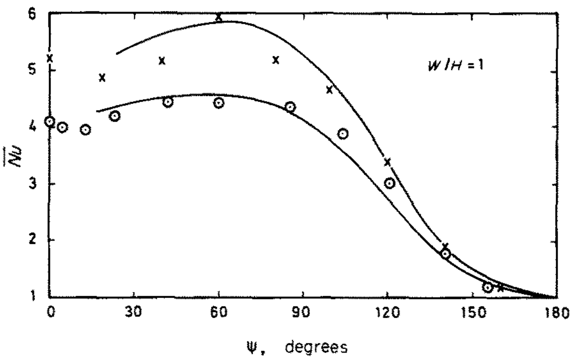

FIG. 2. Effect of angle of inclination on $\overline{Nu}$ for silicone oil and $W/H = 1$. ×, $Q = 4{\cdot}48$ W; $Ra(\text{at }\psi = 0) = 90\,600$, $Pr = 4690$. ⊙, $Q = 2{\cdot}24$ W; $Ra(\text{at }\psi = 0) = 46\,500$, $Pr = 4870$. ——, Theoretical prediction for $Pr = 10$.

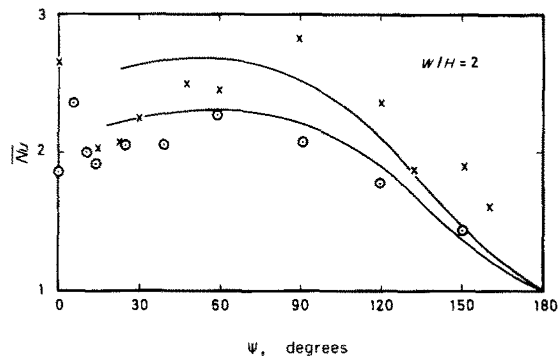

FIG. 3. Effect of angle of inclination on $\overline{Nu}$ for silicone oil and $W/H = 2$. ×, $Q = 4{\cdot}7$ W; $Ra(\text{at }\psi = 0) = 10\,300$; $Pr = 4760$. ⊙, $Q = 2{\cdot}91$ W; $Ra(\text{at }\psi = 0) = 7080$; $Pr = 4860$. ——, Theoretical prediction for $Pr = 10$.

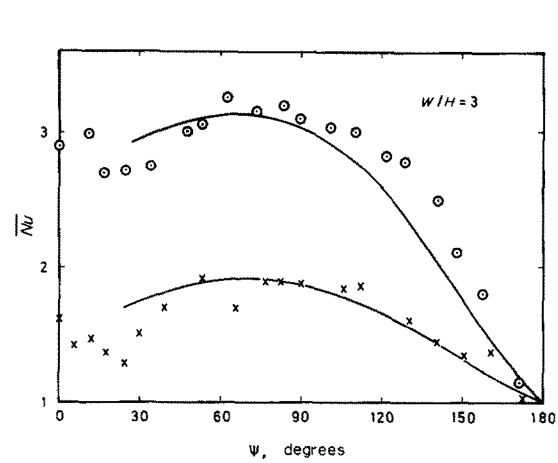

FIG. 4. Effect of angle of inclination on $\overline{Nu}$ for silicone oil and $W/H = 3$. ⊙, $Q = 11{\cdot}92$ W, $Ra(\text{at }\psi = 0) = 20\,000$, $Pr = 3880$. ×, $Q = 2{\cdot}38$ W, $Ra(\text{at }\psi = 0) = 4600$, $Pr = 4270$. ——, Theoretical prediction for $Pr = 10$.

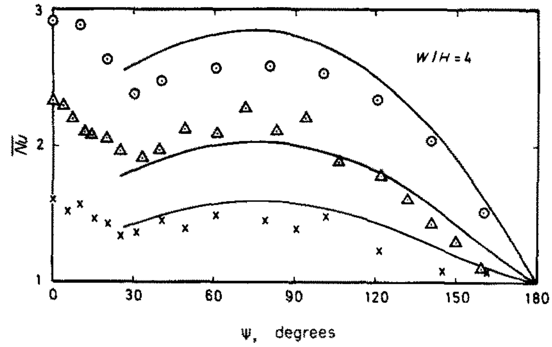

FIG. 5. Effect of angle of inclination on $\overline{Nu}$ for silicone oil and $W/H = 4$. ⊙, $Q = 11{\cdot}76$ W; $Ra(\text{at }\psi = 0) = 16\,800$, $Pr = 3900$. △, $Q = 4{\cdot}71$ W; $Ra(\text{at }\psi = 0) = 6200$, $Pr = 4200$. ×, $Q = 2{\cdot}04$ W; $Ra(\text{at }\psi = 0) = 3050$, $Pr = 4300$. ——, Theoretical prediction for $Pr = 10$.

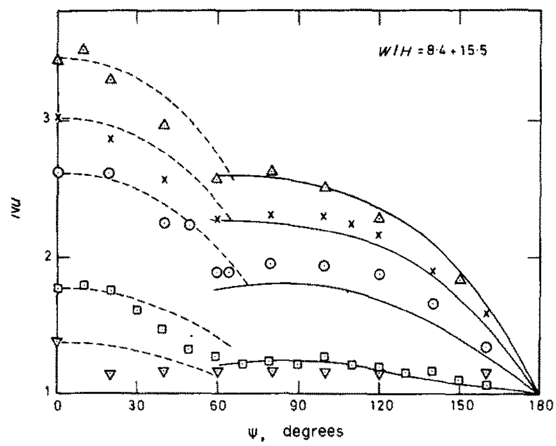

FIG. 6. Effect of angle of inclination on $\overline{Nu}$ for air. △, $Q = 26{\cdot}6$ W; $Ra(\text{at }\psi = 0) = 37\,150$; $W/H = 8{\cdot}4$, $Pr = 0{\cdot}701$. ×, $Q = 13{\cdot}2$ W; $Ra(\text{at }\psi = 0) = 23\,340$; $W/H = 8{\cdot}4$, $Pr = 0{\cdot}701$. ⊙, $Q = 6{\cdot}36$ W; $Ra(\text{at }\psi = 0) = 12\,350$; $W/H = 8{\cdot}4$, $Pr = 0{\cdot}701$. ⊡, $Q = 20{\cdot}08$ W; $Ra(\text{at }\psi = 0) = 3760$; $W/H = 15{\cdot}5$, $Pr = 0{\cdot}701$. ▽, $Q = 7{\cdot}74$ W; $Ra(\text{at }\psi = 0) = 1770$; $W/H = 15{\cdot}5$, $Pr = 0{\cdot}701$. ——, Theoretical prediction for $Pr = 10$ (none for $Q = 7{\cdot}74$ W). ----, Equations (3) and (4).

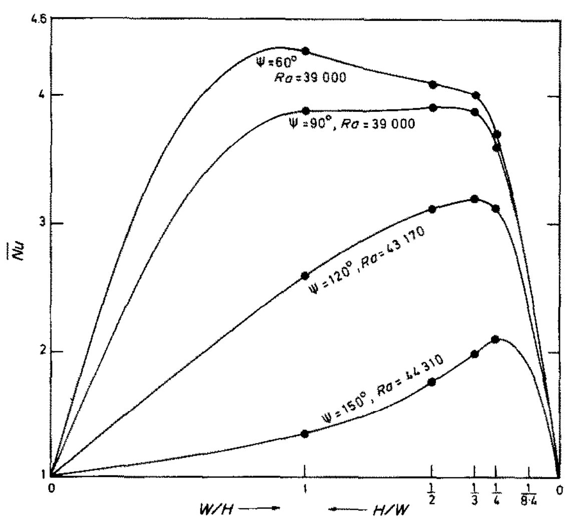

FIG. 7. Computed effect of aspect ratio on $\overline{Nu}$ for $Q = 26{\cdot}6$ W.

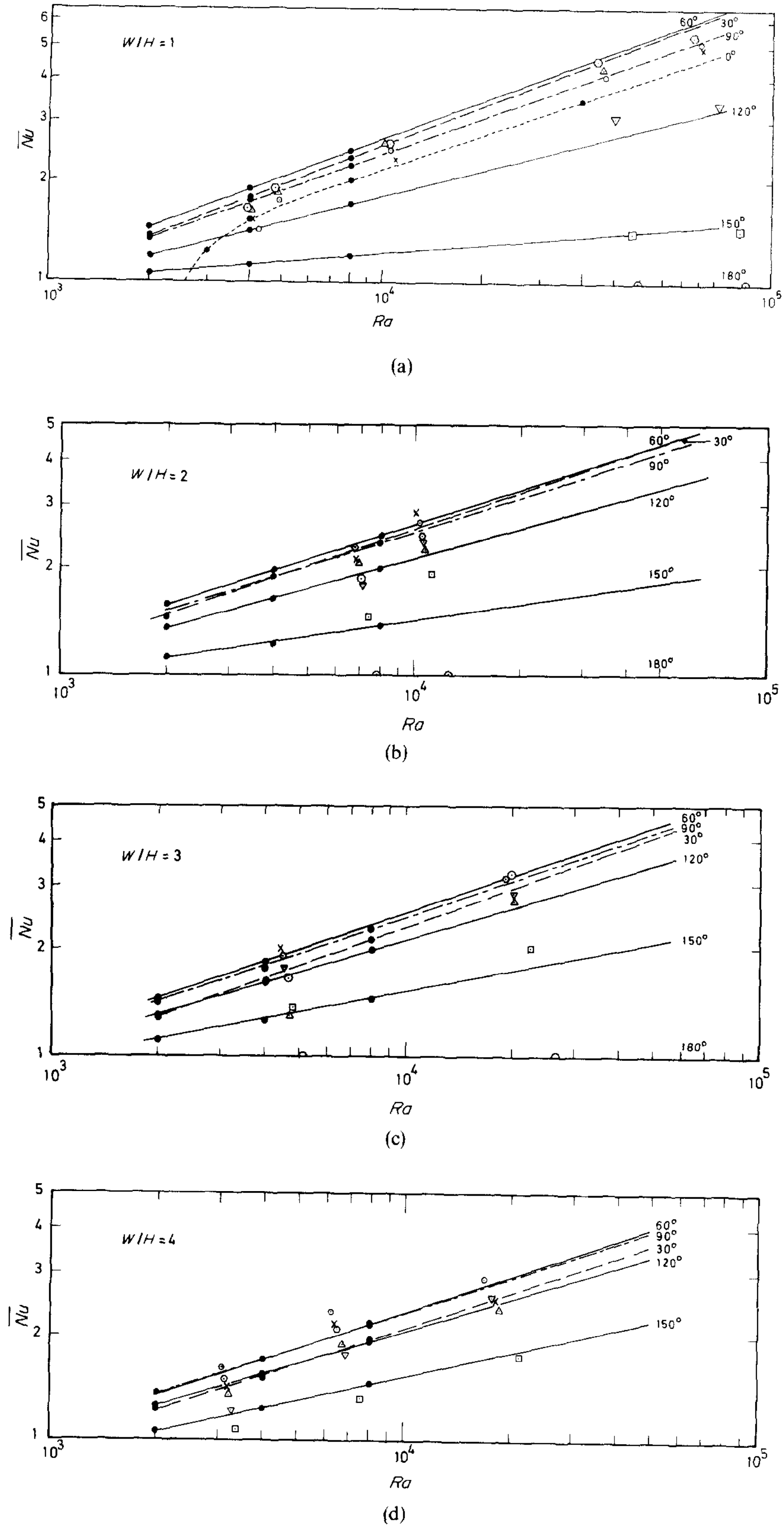

FIG. 8. Effect of Ra on $\overline{Nu}$ for silicone oil

ψ, degrees,	0	30	60	90	120	150	180
Experimental,	⊙	△	⬡	×	▽	⊡	⌒
Theoretical,	——,	— —,	———,	- - - -			

Curves representing the theoretically computed Nusselt numbers are plotted on the same graphs. These curves were constructed for the conditions of the experiments, i.e. uniform surface temperatures but with the temperature difference varied to maintain a constant value of the total heat flux as the angle of inclination was varied. These values were derived from the computed values used to construct Fig. 4 of [9]. The calculations were for aspect ratios of 1, 2, 3, 4 and ∞, $Ra = 2000$, 4000 and 8000. They were only for $Pr = 10$ but the calculations of Samuels and Churchill [10] for rectangular cavities heated from below suggest that these values should be reasonable approximations for $Pr \geqslant 0{\cdot}7$. The theoretical values were found to lie along straight lines when plotted as log $\overline{Nu}$ vs log Ra. The slope and additive constant of each of these lines was determined by least squares. The corresponding constants A and b in the empirical equation

$$\overline{Nu} = ARa^b \qquad (1)$$

are given in Table 2. For 0° the computed values at $Ra = 3000$, 4000 and 8000 did not lie along a straight line. However a computed value of $\overline{Nu}$ at $Ra = 40\,000$ was available for $W/H = 1$ and the constants for equation (1) were computed for $Ra = 8000$ and 40 000 for this case. For this reason the constants in Table 2 for $W/H = 1$ and $\psi = 0$ are proposed for $Ra > 8000$ only while the values for other conditions are proposed for $2000 \leqslant Ra \leqslant 10^5$.

Equation (1) with the constants in Table 2 was used to interpolate and extrapolate the computed values over the range of the experimental measurements. Graphs such as Fig. 7 were then constructed to permit

Table 2. Empirical constants for equation (1)

W/H	ψ (degrees)	A	b	Range
1	0	0·0981	0·336	$Ra > 8000$ only
	30	0·0735	0·385	$2000 \leqslant Ra \leqslant 10^5$
	60	0·0832	[illegible]	↓
	90	0·0919	0·354	
	120	0·1754	0·253	
	150	0·546	0·0867	
2	30	0·1056	0·345	
	60	0·1377	0·320	
	90	0·1400	0·313	
	120	0·1512	0·287	
	150	0·336	0·1569	
3	30	0·0832	0·360	
	60	0·1136	0·334	
	90	0·1233	0·322	
	120	0·1333	0·300	
	150	0·253	0·194	
4	30	0·0932	0·337	
	60	0·1002	0·341	
	90	0·1075	0·332	
	120	0·1185	0·310	
	150	0·1746	0·236	

than those chosen for the calculations. The agreement between experimental points and theoretical lines in Figs. 2–6 is seen to be generally good for the angles of inclination above the maximum in $\overline{Nu}$. At lesser angles of inclination (theoretical curves not shown) agreement was not to be expected due to the change in the mode of circulation.

The data and curves representing the computed and experimental values are also plotted in Fig. 8 as mean Nusselt number vs Rayleigh number with the angle of inclination as a parameter. The data for each channel are plotted on a separate graph to avoid confusion. The various sets of data indicate the variation of Ra with angle of inclination at constant heat input. The agreement between the theoretical and the experimental results in Figs. 2–6 and 8 suggests that the extrapolated theoretical results may be used for prediction up to $Ra = 10^5$. The agreement in Fig. 6 is particularly impressive since the theoretical curves were obtained from the least certain region of Fig. 7.

The experimental angles of inclination at the minimum and maximum values of $\overline{Nu}$ are plotted vs the inverse of the aspect ratio in Fig. 9. The angles for the interpolation and extrapolation for aspect ratios other

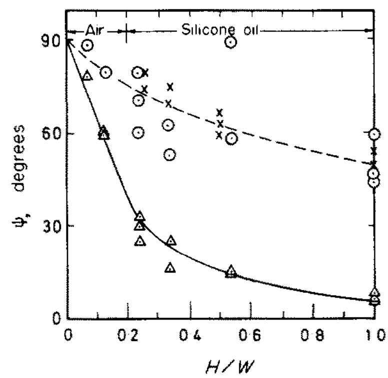

FIG. 9. Effect of aspect ratio on angle of inclination at maximum and minimum values of $\overline{Nu}$. ⊙, Experimental for $\overline{Nu}$-max; ×, Theoretical for $\overline{Nu}$-max; △, Experimental for $\overline{Nu}$-min.

maximum value of $\overline{Nu}$ read from the theoretical curves are also included as discrete points. The solid and dotted lines were drawn for interpolation. This graph suggests that the angles for the maximum and minimum $\overline{Nu}$ converge to 90 degrees as the ratio H/W approaches zero for all of the Rayleigh numbers studied. In the region below the solid curve the preferred mode of circulation is a series of roll cells and $\overline{Nu}$ decreases with angle of inclination. In the region between the solid and dashed curves the preferred mode is a single longitudinal roll cell and $\overline{Nu}$ increases with angle. Above the dashed curve the single roll cell persists but $\overline{Nu}$ decreases with increasing angle.

Values of $\overline{Nu}$ for small angles of inclination and aspect ratios of 8·4 and 15·5 are plotted in Fig. 10 in the form suggested by Clever [5]. These values can be represented by the empirical equation

$$\overline{Nu} = C(Ra\,.\cos\psi)^n. \qquad (2)$$

The line in Fig. 10 has a slope of approximately 1/3. The value of C corresponding to $n = 1/3$ is 0·109. The data for $\psi = 0$ can then be expressed as

$$\overline{Nu}(0) = 0{\cdot}109 Ra^{1/3}$$

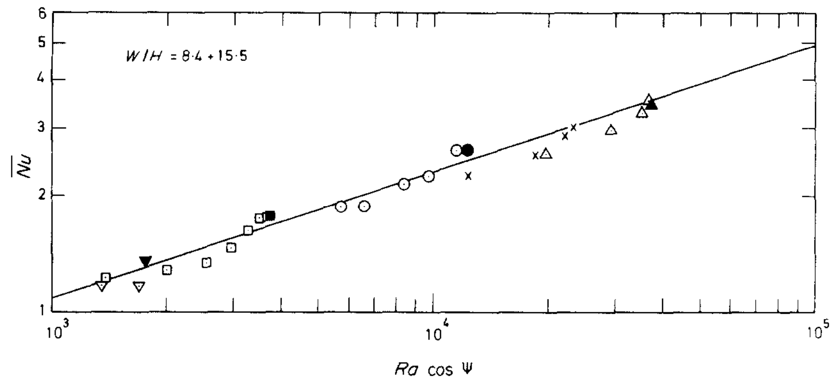

FIG. 10. Empirical correlation for air at low angles of inclination; solid points: $\psi = 0$.

and equation (2) rewritten as

$$\overline{Nu} = \overline{Nu}(0)(\cos\psi)^{1/3}. \quad (4)$$

The dotted lines in Fig. 6 represent equations (3) and (4). The agreement is good but the prediction is consistently high near the minimum value of $\overline{Nu}$, perhaps because of a further change in mode of circulation.

CONCLUSIONS

The experimental rates of heat transfer suggests that the theoretical predictions of [9] can be extended up to 10^5 and an aspect ratio of at least 8·4.

Angles of inclination both at the maximum and the minimum values of $\overline{Nu}$ were correlated as a function of aspect ratio independent of the Rayleigh number, and were found to converge to 90 degrees for infinite flat plates.

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CONVECTION NATURELLE DANS UN CANAL RECTANGULAIRE INCLINE POUR DIVERS RAPPORTS DE SECTION ET ANGLES D'INCLINAISON—MESURES EXPERIMENTALES

Résumé—On a mesuré les taux de transfert de chaleur par convection naturelle laminaire dans l'huile de silicone et dans l'air à l'intérieur d'un long canal rectangulaire. Le rapport de la section droite (largeur/hauteur) du canal a les valeurs 1; 2; 3; 4,2; 8,4 et 15,5 et le nombre de Rayleigh varie de 3.10^3 à 10^5. Le canal est chauffé par dessous et refroidi par dessus tandis que les deux autres côtés sont isolés. La rotation du canal est effectuée par pas autour du grand axe, sur 180 degrés. L'effet de l'inclinaison et du rapport de section sur le taux de transfert thermique a été mesuré expérimentalement.

Lorsque l'angle d'inclinaison varie de 0 à 180 degrés, le taux de transfert de chaleur passe par un minimum. On a trouvé que l'angle d'inclinaison correspondant à ces conditions critiques dépendait fortement du rapport de la section mais peu du nombre de Rayleigh. Une transition dans le mouvement de convection a lieu pour l'angle correspondant au minimum du taux de transfert thermique.

NATÜRLICHE KONVEKTION IN EINEM GENEIGTEN RECHTECKIGEN KANAL BEI VERSCHIEDENEN ANORDNUNGEN UND NEIGUNGEN—EXPERIMENTELLE MESSUNGEN

Zusammenfassung—Für laminare natürliche Konvektion in Silikon-Öl und Luft in einem langen rechteckigen Kanal wurden Wärmeübergangs-Messungen durchgeführt. Das Anordnungsverhältnis (Breite/Höhe) eines Kanal-Querschnitts wurde variiert mit 1; 2; 3; 4,2; 8,4 und 15,5 und für Rayleigh–

Zahlen von 3×10^3 bis 10^5. Der Kanal war von unten beheizt und von oben gekühlt. Die beiden Seiten waren isoliert. Der Kanal wurde um die lange Achse über 180° schrittweise geneigt. Der Einfluß der Neigung und des Anordnungsverhältnisses auf den Wärmeübergang wurde experimentell bestimmt.

Mit ansteigendem Neigungswinkel von 0 bis 180° wurde ein Minimun und ein Maximum im Wärmeübergang festgestellt. Der Neigungswinkel bei diesen kritischen Bedingungen erwies sich als stark abhängig vom Anordnungsverhältnis und schwach abhängig von der Rayleigh–Zahl. Ein Übergang in der Zirkulationsform ergab sich bei einem Winkel, der einem minimalen Wärmeübergang entsprach.

ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ В НАКЛОННОМ ПРЯМОУГОЛЬНОМ КАНАЛЕ ПРИ РАЗЛИЧНЫХ ОТНОШЕНИЯХ ШИРИНЫ КАНАЛА К ВЫСОТЕ И РАЗЛИЧНЫХ УГЛАХ НАКЛОНА

Аннотация — Измерялся теплообмен при ламинарной естественной конвекции в силиконовом масле и в воздухе в длинном прямоугольном канале. Величина отношения ширины канала к высоте изменялась как 1; 2; 3; 4,2; 8,4; и 15,5; число Релея изменялось от 3×10^3 до 10^5. Канал нагревался снизу и охлаждался сверху, а две другие его стороны были изолированы. Канал вращался вокруг длинной оси постепенно до 180°. Экспериментально определялось влияние угла наклона и отношения ширины канала к высоте на теплообмен.

Минимальное и максимальное значение теплообмена наблюдалось, когда угол наклона увеличивался от 0 до 180°. Найдено, что в таких критических условиях угол наклона является сильной функцией отношения ширины канала к высоте и слабой функцией числа Релея. Изменение структуры конвективного движения происходило при значении угла наклона, соответствующем минимальному теплообмену.